What strangeness tells us about the hadronic equation of state

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The nucleus as a laboratory:
Strangeness production at threshold determines the nuclear equation of state
The nucleus allows to determine the KN interaction and its density dependence

IWARA 2009, Maresias, Brasil 2009
Why is this relevant for astrophysics?

Equation of state appears in the calculations of neutron stars or binaries properties

Properties of those stars depend crucially on properties of strange hadrons in matter

Kaon properties in (proto-)neutron star matter
A. Mishra, A. Kumar, S. Sanyal, V. Dexheimer, S. Schramm
arXiv:0905.3518 [nucl-th]

The Quest for the Nuclear Equation of State.
J. Aichelin, Jürgen Schaffner-Bielich
arXiv:0812.1341 [nucl-th]
How to get this information from experiments?

Not directly: none of the observables give direct information

But by comparison with theory

One simulates the complete heavy ion reaction on the computer using different eos and different KN potentials and compares the results with experiment

Problem: there are many little known or unknown (in medium) cross sections and other not well determined quantities (lifetimes of resonances, potential ranges etc)

-> One has to assure that the observables are robust
Our tool: IQMD

Isospin-Quantum Molecular Dynamics model

- Semiclassical dynamical N-body model with quantum features based on 2- and 3-body interactions

- Microscopic calculation of heavy ion collisions on an event-by-event-basis

- includes N, Δ, π with isospin d.o.f.

- strange particles treated virtually

Allows for a «photo» of the high density phase and for a look inside …
Simulation of a collision Au+Au @ 1.5 AGeV b=0 with IQMD

红: 质子         灰: 中子         绿: Δ粒子        蓝: π粒子
The original idea of measuring the eos

- Eos describes the energy needed to compress nuclear matter.

- For a given density, a hard eos requires more energy than a soft one to compress matter.

- For a given density and a given available energy, a soft eos leaves more thermal energy to the system than a hard eos.

- R. Stock: This thermal energy could be measured by regarding pion production.
Is this idea true?

Hard and soft eos reach different maximum densities and the pion numbers are only slightly different. For a small system the differences in density vanish. The differences in pion yield as well.

However the kaons show significant differences.
Subthreshold kaon production

• Production of kaons at energies below the kinetic threshold for K production in elementary pp collisions

• Fermi momenta may contribute in energy

• Multi-step processes (especially with a $\Delta$) can cumulate the energy needed for kaon production. Short $\Delta$ lifetime enhances production at high densities

• Repulsive KN potential cause a penalty factor at high density

• The $\rho$ dependence of both yield a sensitivity to in-medium effects and nuclear equation of state
Kaons test high densities

Multistep processes require high densities, but medium effects of kaons penalize the high density production.

Effect of repulsive KN potential and multistep processes compensate to a large extend but sensitivity to the eos still survives. However, the absolute yield depends on many details and is not conclusive.
Many combinations are possible - Many channels have to be implemented

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- Each channel contains isospin subdivisions
- Only few channels (like pp$\rightarrow$p$\Lambda$K$^+$) are measured by the experiment (even incomplete infos)
- Significant incertainties from parametrization of unknown channels or isospin subdivisions
Eos cannot be deduced directly from kaon yields! 
Incertainties of cross sections larger than eos effect 

However, the eos effect vanishes for small A while 
the cross section effect persists up to small A.
The solution: use ratios \( \text{Au/C} \)
This cancels many uncertainties

Data: KaoS @GSI

IQMD supports this (although IQMD and RQMD differ in absolute yields)

RQMD: Ch. Fuchs
1:0 for soft
A observation which is robust versus effects of production cross sections, KN-potential, less stopping (reduced $\sigma_{NN}$), lifetime of the $\Delta$, …
Second independent Observable:

**Au: central versus peripheral**

Different cross sections and potential parameters may change the global yield. However, the parameter $\alpha$ for the increase of the kaon yield $N$ with the number $A$ of participating nucleons (raising with centrality)

$$N(K) = N_0 A^{\alpha}$$

depends on the eos. A soft eos yields higher values than a hard eos.
Determination of the eos from $\alpha$

The relation between the compression modulus and $\alpha$ is monotonously falling. KaoS data (Förster et al.) favor a value below 200 MeV, i.e. a soft eos.

2:0 for soft
Energy dependence of the system size systematics

Similar method, now using system size $A$ of inclusive events

$M(K^+) = A^\alpha$

System size

Preliminary

$A_{part}$ in Au+Au agrees with that

3:0 for soft

Soft eos confirmed
Consequences for neutron stars:


Radius mass relation of EOS compatible with heavy ion results
Eos compatible with HI data but with different extensions to high density (which determines the maximal mass)

All observed stars are compatible with this.
Can we measure the $K^+$ and $K^-$ potential directly?

**Problem:** almost all observables depend simultaneously on several input quantities, like cross sections, in medium life times etc.

Therefore we have to find **observables which are (almost exclusively) dependent** of the KN potential.

These observables exist!!

but are different $K^+$ and for $K^-$ and (unfortunately) more precise for the $K^+$ than for $K^-$. 
**First condition**: Data have to be reproduced in the simulation programs

Experimental results from KAOS@GSI confronted to IQMD
The potential K+ nucleon interaction is repulsive
  -> the K+ mass increases with density
  -> between production and detection the K+ has to get rid of
    the mass excess by converting mass in kinetic energy
  -> mass excess is proportional to the potential
  -> additional momentum should be proportional to the energy
  -> modifications should be most visible at small pT

Low \( p_t \) K\( ^+ \) are indeed sensitive to the KN potential

\[ U_{KN} = \alpha \times 35 \text{ MeV } \rho/\rho_0 \]

\( \alpha = 1 \) : prediction from nucl. Matter calculations
$U_{KN} = \alpha \cdot 35\text{ MeV} \frac{\rho}{\rho_0}$

**CC/ArAr:**
- different density (CC semi-transpar.)

**ArAr/AuAu:**
- same density

Rescattering $\sigma$
- unimportant (few coll)
- important (changes spectra)
Second condition: the variable has to be robust:

Neither the NN potential nor the (unknown) $\sigma_{N\Delta}$ or modifications of the $\sigma_{NN}$ cross section can change the conclusion. They modify the low $p_T$ ratio in a controllable way.
And what does experiment say?

**HADES@GSI data  1.75 AGeV  Ar + KCL**

Absolute normalization (b < 6 fm)

Normalized to $p_t > 450$ MeV/c

$U(p_\rho) = \alpha \times 35$ MeV

$K^0_S$ (HADES)

IQMD ($\alpha = 0.0$)

IQMD ($\alpha = 0.5$)

IQMD ($\alpha = 0.7$)

IQMD ($\alpha = 1.0$)

IQMD ($\alpha = 1.2$)

IQMD ($\alpha = 1.35$)

IQMD ($\alpha = 1.5$)

$1.0 < \alpha < 1.2$

$\rightarrow$ strength of $U=42\pm 10$ MeV @ $\rho=\rho_0$

KN-Potential is repulsive

predicitons agree (almost) with exp
And what’s about the $K^-$?

**Much more difficult**
- Little known cross sections $\sigma (\pi \Lambda \rightarrow N K^-)$ or unknown cross sections $\sigma (N \Lambda \rightarrow NNK^-)$ involved

- $K^-$ freeze out late (as compared to $K^+$) therefore potential at freeze out smaller (despite of stronger density dependence)

- Form of low $p_T$ spectra depends on (unknown) cross sections as well as on the potential

One has to find other ways to test the potential.
\( \omega(k=0) \) or the 'mass' decreases with density

\( \rightarrow \) when the density becomes large the mass becomes zero

in AuAu collisions the density is always higher than in CC

\( \rightarrow \) as soon as \( \omega(k=0) \) becomes \( >0 \) cross section explodes

One has to measure the multiplicity ratio AuAu/CC as a fct of \( E_{\text{beam}} \) to determine the NK\(-\) potential
Present data ($E_{\text{beam}} < 1.5 \text{ AGeV}$) show a very smooth increase of the multiplicity ratio.

Consequently for

$$U_{\text{KN}} = -\alpha \times 110 \text{ MeV } \rho/\rho_0$$

with $\alpha=1$ being the mean field prediction of Schaffner-Bielich et al. (very conservative)

we find $\alpha < 1.35$.

$\rightarrow$ K$^-$N potential is rather flat

$\rightarrow$ no K$^-$ condensates
Conclusions

K$^+$ data from KaoS@GSI (and FOPI@GSI) (E< 1.5 AGeV) are only consistent with a soft equation of state (K~200 MeV)

- Excitation fct of K$^+$ yield of AuAu/CC
- K$^+$ yield as a fct of participant numbers
- System size dependence of K$^+$ yield at different energies

Neutron star results are compatible with this value

Low momentum p$_t$ spectra: $\omega(k=0)_{K+N}(\rho/\rho_0 =1) = m_0 + 42 +/- 10$ MeV.

- The smooth experimental multiplicity ratio of K$^-$ in AuAu and CC yield $\omega(k=0)_{K-N} (\rho/\rho_0 =1) > m_0 - 150$ MeV

-> K$^-$ condensates are excluded up to densities of 3 $\rho/\rho_0$. 
High density: medium effects

Optical potential: repulsive for $K^+$, enhances its « mass » → Penalty factor for production at high density

Use of Schaffner-Bielich RMF results as standard

$$U^K_{opt} = \sqrt{(\vec{k} - g_\sigma \vec{\Sigma}_\sigma)^2 + m_K^2 + m_K g_s \Sigma_s + g_\sigma \Sigma_v^0 - \sqrt{k^2 + m_K^2}}$$
Kaons and density isomers

- Could reveal density isomers by a sudden rise in the excitation function of kaons - KaoS might measure it

A 2\textsuperscript{nd} minimum would yield a sudden factor of 10 in the kaon yield

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Subthreshold Kaons Would Reveal Density Isomers

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(Received 22 September 1993; revised manuscript received 7 April 1994)
A density isomer would have needed the strong raise indicated by the arrows. IQMD calculations using a KN optical potential and a soft eos are consistent with KaoS data on Au+Au and C+C of Sturm et al.

**KaoS DATA**: no isomer up to $3\rho_0$

A density isomer would have needed the strong raise indicated by the arrows.

**IQMD calculations using a KN optical potential and a soft eos are consistent with KaoS data on Au+Au and C+C of Sturm et al.**
Analysis at lower beam energy

A soft equation of state is favoured.

Acceptation range for $K$. 
Going down in beam energy

A soft eos yields $\alpha \approx 1.4$ at $E=0.8$ AGeV, a hard eos yields $\alpha \approx 1.2$

Limits for lower $E$: no asymptotic yield for peripheral collisions
A soft eos obtains higher kaon yields for heavy systems.

KaoS: PRC in preparation
Definition of the potentials

\[ V_{ij} = G_{ij}^{i} + V_{Coul}^{i,j} \]

\[ = V_{Skyrme}^{ij} + V_{Yuk}^{ij} + V_{md}^{ij} + V_{Coul}^{ij} + V_{sym}^{ij} \]

\[ = t_1 \delta(x_i - x_j) + t_2 \delta(x_i - x_j) \rho^{-1}(x_i) + t_3 \frac{\exp\left\{ -|x_i - x_j|/\mu \right\}}{|x_i - x_j|/\mu} + \]

\[ t_4 \ln^2(1 + t_5 (\vec{p}_i - \vec{p}_j)^2) \delta(x_i - x_j) + \]

\[ \frac{Z_i Z_j e^2}{|x_i - x_j|} + \frac{1}{\rho_0} T_i^3 T_j^3 \delta(\vec{r}_i - \vec{r}_j) \]

2 and 3 body interactions
(no equilibrium required)

Bethe Weizsaecker –mass formula:

Volume term +Surface term +Coulomb term +symmetry term
(with eos) (+pairing term not included)
The static part (our «eos»)

3 parameters, 2 ground state condit.

1 remaining d.o.f.: compression mod.

Artificial link between curvature at ground state and high density behaviour.

Compression modulus $K > 170$ MeV

Problems of causality for high densities $\rho > 5-7 \rho_0$

Caution when extrapolating to high densities
The eos in IQMD after the convolution of the Skyrme type potentials supplemented by momentum dependent interactions (mdi) for infinite saturated nuclear matter at equilibrium.

\[ U = \alpha \cdot \left( \frac{\rho_{int}}{\rho_0} \right) + \beta \cdot \left( \frac{\rho_{int}}{\rho_0} \right)^\gamma + \delta \cdot \ln^2 \left( \varepsilon \cdot (\Delta \vec{p})^2 + 1 \right) \cdot \left( \frac{\rho_{int}}{\rho_0} \right) \]

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<th>( \beta ) (MeV)</th>
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<th>( \delta ) (MeV)</th>
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Different densities and different pressure

Next idea on eos: do not use the compressional energy but the repulsion of the potential  

Nucleonic flow
In-plane flow, Squeeze, pion flow

Test of density gradient and geometry

Transverse flow dominated by «cold » matter

Dense matter tends towards isotropy

Pion flow: test on resonance matter

Comparison of Plasticball squeeze favors soft eos+mdi

For recent analysis on FOPI data see the contributions of Willi Reisdorf and Anton Andronic