On magnetized SQM and magnetized SQS

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Outline

1. Introduction.
2. Magnetized SQM
3. B/MIT Bag Model
4. Real scenario: Numerical Results
   - Stability MSQM
   - EOS-MSQM
5. Mass-Radius Relation/MSSs.
6. Conclusions
7. Other works...
Strange Quark Matter/Strange Stars

1. SQM means quarks up (u), down (d) and strange (s) deconfined. It appears in extreme condition! Two extreme scenarios:

2. High T: Early Universe (QGP) $(T \sim 10^{12} K \sim 150 \text{Mev})$; In some experiments like heavy ions collisions (RHIC)

3. Ultrahigh density: Inner of neutrons stars or properly strange stars (density $2.5 - 5 \times n_0$)

SQSs composed by SQM was suggested by Itoh in 1970. It was based on Conjecture of the Bodmer-Witten-Terasawa (1970-1984) about the existence of SQM.
Stability of SQM: E/A

Conjecture of BWT: At $P = 0$, $T = 0$ and finite density the $\frac{E}{A}|_{SQM} < \frac{E}{A}|_{^{56}Fe}$:

$$\frac{E}{A}|_{SQM} = 829 \text{MeV}$$

$$\frac{E}{A}|_{^{56}Fe} < \frac{E}{A}|_{u,d} < m_n$$

SQM is the ground state of the matter.
### Strange Stars/NSs

Differences between NSs/SSs

\[
\begin{align*}
\text{M–R} & \quad \begin{cases} 
\text{NSs} & \rightarrow M \propto R^{-3} \\
\text{SQSs} & \rightarrow M \propto R^3
\end{cases}
\end{align*}
\]

- SQs are self-bounded (due to the strong interaction) NSs Gravity is the responsible.
- SQSs can rotate more faster than NSs.
What observations could be SSs?/NASA 2002

Candidates: NASA 2002

- Enigmatic source of X-rays RX J1856.5-3754. Radio (3.8-8.2 Km) little to be a NS
- Q3C58 Remanent of the SN seen in 1181 T at the surface < than NS
- Remanent of the SN1987A ?

Figure: Remanent of the Supernova 1987A?
Why we want to introduce the effect of magnetic field?

- Astrophysical motivation: It is a fact that Magnetars have magnetic fields around $10^{15}$ at the surface and inside are possible $B$ up to $10^{18}$ G or may be more higher.

- Magnetized compact objects could explain intense $\gamma$ repeaters and sources of X rays.

- Particle physics motivation: In heavy ions colliders, there are strong magnetic field at least locally.

  We know that microscopic properties of magnetized hadron/quark matter exhibit exotic properties:

  - softer EOS for the system.

  - This implies to obtain more compact stable configurations of SSs.
Phenomenological Models to study EOS

EOS are obtained starting from the phenomenological model:

- MIT Bag Model is one of the most important model using for that. In MIT Bag model the quarks are treated as a degenerate Fermi gas confined to a region of space having a vacuum energy density $B_{bag}$ (the bag constant) the confinement is guarantied by the bag parameter.

- Nambu Jona Lasinio Model (NJL) which is chiral invariant but it not reproduces the confinement.
MIT- Bag model general description

\[ P = -\Omega_T, \quad \epsilon = \Omega + \sum_f \mu_f N_f + TS \]

\[ \Omega_T = B_{\text{bag}} V + \Omega, \quad B_{\text{bag}} = \Omega_{\text{vac}} \]

\[ \Omega = \sum_f \left[ \Omega_{f,v}(\mu_f, T)V + \Omega_{f,s}(\mu_f, T)S + \Omega_{f,c}(\mu_f, T)C \right], \]

\( f = (e, u, d, s) \), quarks and electrons. Vacuum:Bag, guarantied confinement;

\[ \Omega_{f,v}(\mu_f, T) = -\frac{d_f T}{(2\pi)^3} \int \ln[1 + \exp\{-\frac{E_{p,f} - \mu_f}{T}\}] d^3 p, \]

for to get EOS the surface terms are neglected, they are important for study transition phase: nucleation

\[ \Omega_{f,s}(\mu_f, T) = \frac{d_f T}{64\pi^2} \int [G_\sigma \ln[1 + \exp\{-\frac{E_{p,f} - \mu_f}{T}\}] d^3 p] \quad G_\sigma(p) = \{1 - \frac{2}{\pi} \tan^{-1}\left[\frac{\vec{p}}{m_f}\right]\} \]

\[ \Omega_{f,c}(\mu_f, T) = \frac{d_f T}{48\pi^3} \int [G_\gamma \ln[1 + \exp\{-\frac{E_{p,f} - \mu_f}{T}\}] d^3 p] \quad G_\gamma(p) = \{1 - \frac{3}{2m_f} \left\{\frac{\pi}{2} - \tan^{-1}\left[\frac{\vec{p}}{m_f}\right]\right\} \} \]
In the degenerate case with massless quarks we have

\[ \epsilon_q = \frac{\mu_q^4}{(4\pi^2(hc)^3)}, \quad P_q = \frac{\mu_q^4}{(4\pi^2(hc)^3)}, \quad N_q = \frac{\mu_q^3}{3\pi^2(hc)^3}, \]

\[ \mu_q \text{ chemical potential of quarks (u,d,s)} \]

\[ P + B_{bag} = \sum_q P_q \quad \epsilon - B_{bag} = \sum_q \epsilon_q \]

**EOS**: \[ P = \frac{(\epsilon - 4B_{bag})}{3} \]
What we study?

- The Stability of MSQM- $E/A$ ?

$$\frac{E}{A}_{u,d,s}^B < \frac{E}{A}_{u,d,s}^{B=0} < \frac{E}{A}_{56\text{Fe}} < \frac{E}{A}_{u,d}^B < \frac{E}{A}_{u,d}^{B=0} < m_n,$$

- The EOS of MSQM
- To get stable configurations of non-rotating MSSs
- We consider a constant magnetic field in the $x_3$ direction.
- We take into account the role of AMM-for quarks.

To do that we use the MIT Bag Model.
Introduction to B/MIT Bag Model

Our aim is to introduce a magnetic field $B$: in general we put $\mathcal{F}_{\mu \nu}$ in the Lagrangian of MIT Bag Model

$$\mathcal{L}_{bag}(\mathcal{F}_{\mu \nu}) \Rightarrow T_{\mu \nu}^{bag}(\mathcal{F}_{\mu \nu})$$

$$\downarrow$$

$$B_{bag} = -\frac{1}{2} n_i \left( \frac{\partial}{\partial x_i} \psi(x) \psi(x) \right)$$

$$\downarrow$$

$$\ll T_{\mu \nu}^{bag}(\mathcal{F}_{\mu \nu}) \gg$$

conclusion: $B_{bag}$ in the presence of a magnetic field has an anisotropic $\Rightarrow$ depends on the $B$-direction in space, here related to the $n_i$ direction.
Anisotropy due to B

If we have B constant in the $x_3$-direction. The energy momentum tensor in presence of magnetic field has the form

$$\mathcal{T}_{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P_\perp & 0 & 0 \\ 0 & 0 & P_\perp & 0 \\ 0 & 0 & 0 & P_\parallel \end{pmatrix}$$

where

$$P_\perp = -\Omega - MB \quad P_\parallel = -\Omega, \quad B = 0 \quad P_\parallel = P_\perp$$

Calculations:

In order to get EOS we have to calculate the following quantities

- $\Omega(\epsilon_n) E_n$ depends on $B$, $n$-Landau levels, $\mu_i$, $m_i$, $y_i$-AMM moment.
- $\mathcal{M} = -\partial \Omega / \partial B$
- $N = -\partial \Omega / \partial \mu$
- $P_\perp = -\Omega - \mathcal{M} B,$
- $\epsilon = \Omega + \mu N$
Energy-AMM/Ground state of particles

Spectrum of particles

\[ E_{i,n}^{\eta} = m_i \sqrt{x_i^2 + \left( \frac{B}{B_c^i} (2n + 1 - \eta) + 1 - \eta y_i B \right)^2} \]

\[ B_c^i = \frac{m_i^2}{|e_i|}, \quad x_i \equiv \frac{p_3}{m_i}, \quad y_i = \frac{|Q_i|}{m_i}, \quad i = (e, u, d, s), \]

<table>
<thead>
<tr>
<th>i</th>
<th>e</th>
<th>u</th>
<th>d</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>m_i</td>
<td>0.5Mev</td>
<td>5Mev</td>
<td>5Mev</td>
<td>150Mev</td>
</tr>
<tr>
<td>Q_i</td>
<td>0.00116\mu_B</td>
<td>1.85\mu_N</td>
<td>-0.97\mu_N</td>
<td>-0.58\mu_N</td>
</tr>
</tbody>
</table>

Ground state \((n = 0, \eta = 1, )E_{i,0} = m_i(1 - y_i B)\)

Appearance of a threshold value for the magnetic field at which the effective mass vanishes, \(m_i \sim |Q_i| B\). \(B_e^s = 7.6 \times 10^{16} \text{ G}, \quad B_u^s = 8.6 \times 10^{17} \text{ G}, \quad B_d^s = 1.6 \times 10^{18} \text{ G}, \quad B_s^s = 8.2 \times 10^{19} \text{ G}\)
Magnetized SQM (MSQM)

\[ \epsilon = B \sum_i \mathcal{M}_i^0 \sum_n \left( x_i g_i^\pm + h_i^\pm 2 \ln \frac{x_i + g_i^\pm}{h_i^\pm} \right). \]

\[ P_{\parallel} = B \sum_i \mathcal{M}_i^0 \sum_n \left( x_i g_i^\pm - h_i^\pm 2 \ln \frac{x_i + g_i^\pm}{h_i^\pm} \right) \]

\[ P_{\perp} = B \sum_i \mathcal{M}_i^0 \sum_n \left( 2 h_i^\pm \gamma_i^\pm \ln \frac{x_i + g_i^\pm}{h_i^\pm} \right). \]

\[ N = \sum_i N_i^0 B_i B_c^0 \sum_n g^\pm, \quad N_i^0 = \frac{d_i m_i^3}{2\pi^2}. \]

Anisotropy of the Bag due to the magnetic field

\[ g_i^\eta = \sqrt{x_i^2 - h_i^\eta^2} \quad \text{magnetic Fermi energy of particles} \]

\[ h_i^\eta = \sqrt{\frac{B}{B_c^i} (2n + 1 - \eta) + 1 - \eta y_i B} \equiv m_i^B \quad \text{magnetic mass of the particles} \]
Stability condition of the SQM

\[ P_T + B_{bag} = \sum_i P_i \quad \varepsilon - B_{bag} = \sum_i \varepsilon_i \]

under the condition \( P = 0 \).

Magnetization is always a positive function, the anisotropy in the pressures implies

\[ P_\perp < P_\parallel. \]

Thus, the stability condition for strong fields changes from to \( P_\perp = 0 \).

Thus, the stability condition for strong fields changes from \( P = 0 \) to \( P_\perp = 0 \) or, equivalently,

\[ P_\perp = \sum_i P_{\perp,i} - B_{bag} = 0. \] (1)

As it turns out from our numerical analysis (see figures below), in this case the total energy \( \varepsilon \) given is always lower than \( 4B_{bag} \) and, therefore, MSQM is more stable than non-magnetized SQM.

For fields \( B < 10^{18} G \) \( P_\perp \approx P_\parallel \)
SQM (core of NSs or SSs) is determined by a set of equilibrium conditions given by:

\[
\mu_u + \mu_e = \mu_d, \quad \mu_d = \mu_s,
\]
\[
2N_u - N_d - N_s - 3N_e = 0,
\]
\[
\frac{1}{3}(N_u + N_d + N_s) = n_B .
\]

fixing the value of the B and using the EOS derived by

\[
P_{\perp} + B_{bag} = \sum_q P_q
\]

we have 4 eq and 4 variables to determine (all chemical potentials and \( B_{bag} \))
Results: fixing $n_B = 0.4 \text{ fm}^{-3}$.

Density versus B

Anisotropy of pressures

N versus B

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Stability of SQM/magnetic field

**Figure:** E/A versus $n_B/n_0$, $ud + e$ and SQM

**Figure:** E/A versus Pressure, $ud + e$ and SQM

MSQM most stable!
Figure: SQM in the plane $(m_s - n_B)$ for $B = 0$

Figure: SQM $B = 5 \times 10^{18} G$. B moves the windows
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Figure: EOS for $B = 0$, $B = 5 \times 10^{18} G$
EOS-MSQM, $ud + e$

Figure: EOS for $B = 0, \ B = 5 \times 10^{18} G, \ B = 10^{19} G$
Solving the Tolman Oppenheimer Volkoff (TOV) for

\[
\frac{dM}{dr} = 4\pi Gr^2 \epsilon(r)
\]

\[
\frac{dP}{dr} = -G \frac{(\epsilon(r) + P(r))(M + 4\pi P(r)r^3)}{r^2 - 2rM(r)}
\]

with the boundary conditions \( M(0) = 0, \ P(R) = 0 \) and the EOS obtained numerically to determine \( P(0) = P_c \).

It is possible to determine configurations of spherical symmetric non-rotating compact stars.
Solving the TOV for $B = 0, 5 \times 10^{18} G, 10^{19} G$.

$B (G)$ | $M_{\text{max}} / M_{\odot}$ | $R (km)$  
---|---|---
0 | 1.87 | 10.35  
$5 \times 10^{18}$ | 1.84 | 10.16  
$10^{19}$ | 1.76 | 9.73

$B (G)$ | $R_{\text{max}} (km)$ | $M / M_{\odot}$  
---|---|---
0 | 10.80 | 1.73  
$5 \times 10^{18}$ | 10.60 | 1.69  
$10^{19}$ | 10.17 | 1.58

$M_{\text{max}} (R_{\text{max}})$ and the corresponding R (M) for MSQS for B and $B_{\text{bag}} = 57 \text{ Mev} / \text{fm}^3$. Blue shaded areas correspond to regions of confidence contours for the MR of the X-ray binary EXO 1745-248 (F. Özel et al Astrophys J 693 1775 (2009))
Conclusions

- Stability of MSQM using the phenomenological MIT Bag-Model was investigated. We take into account variation of all the parameters of the model: mass of the q-strange, baryon density, magnetic field, Bag parameter. Magnetic field moves all the space of parameters.

- For SQM the stability range for the baryon density is $1.8 \lesssim n_B/n_0 \lesssim 2.4$ for $50 \leq m_s \leq 300$ MeV, MSQM allows densities in the range $1.85 \lesssim n_B/n_0 \lesssim 2.6$ for $50 \leq m_s \leq 240$ MeV and a magnetic field value of $5 \times 10^{18}$ G. Moreover, the allowed range for the bag parameter is $57 \lesssim B_{bag} \lesssim 90$ MeV/fm$^3$.

- MSQM is more stable than SQM no-magnetized.

- We have obtained configurations of stable magnetized SQS with mass and radius lower than the obtained for SQM.

Conclusions

It is true!!!

\[
\frac{E}{A} \bigg|_{SQM}^{B} < \frac{E}{A} \bigg|_{SQM}^{B=0} < \frac{E}{A} \bigg|_{56\text{Fe}}^{u,d} < \frac{E}{A} \bigg|_{u,d}^{B} < \frac{E}{A} \bigg|_{u,d}^{B=0} < m_n,
\]
## Others works

| Study of CFL in presence of magnetic field (in progress...in collaboration with D. Manreza Paret and R. González Felipe), almost finished |
| Study the transition phase for quark-hadron and nucleation in collaboration with Ernesto Lopez Fune: effects of Temperature, density and magnetic field, almost finished |
| Study of strangelets in presence of magnetic field |
Collaborators
Quark matter using NJL SU(2)

This work was a consequence of iwara07: It was studied the quark matter in presence of magnetic field using Nambu-Jona-Lasinio-SU(2) (D. P. Menezes, M. Bengui Pinto S. S Avancini, A. Pérez Martínez and C. Providença (PHYSICAL REVIEW C 79, 035807 (2009)))

FIG. 4. (a) Binding energy and (b) EOS for different values of the magnetic field for $ud$ matter with equal chemical potentials, $B_0 = 10^{19}$ G.
Thanks, Obrigada, Gracias !!!