Nonlocal chiral quark models with Polyakov loop at finite $T$ and $\mu$

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PLAN OF THE TALK

• Introduction

• Non-local chiral quark models

• Two flavor non-local models with Polyakov loop

• Outlook & Conclusions

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A type of model that has recently received attention is the Polyakov-Nambu-Jona-Lasinio model: Fukushima (03), Megias, Ruiz Arriola, Salcedo (06), Ratti, Thaler, Weise (06),...

NJL model (CHIRAL DYNAMICS)

Synthesis of

• NJL model is the most simple and widely used model with chiral quark interactions. Local scalar and pseudoscalar four-fermion couplings + regularization prescription (ultraviolet cutoff)

NJL (Euclidean) action

\[ S_E = \int d^4x \left\{ \bar{\psi} \left( -i\gamma^\mu \partial_\mu + m_c \mathbb{1} \right) \psi - \frac{G}{2} \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2 \right] \right\} \]

Nambu, Jona-Lasinio, PR (61)

• Polyakov loop

Polyakov, PLB (78)

\[ \Phi(\vec{x}) = \frac{1}{N_c} Tr \left[ i \int_0^{\beta} d\tau A_4(\vec{x}, \tau) \right] \]

\[ U(\Phi) \] Effective potential

\( U(\Phi) \)

confinement: Z(3) symmetry not broken

\[ \langle \Phi \rangle = 0 \]

\( T < T_c \)

pure gauge \( \rightarrow \) Z(3) symmetry

\[ \Phi \rightarrow z \Phi \]

\[ z = \exp \left[ i \frac{2\pi n}{N_c} \right] \]

\( n = 0, 1, \ldots, N_c - 1 \)

deconfinement: Z(3) symmetry spontaneously broken

\[ \langle \Phi \rangle \neq 0 \]

\( T > T_c \)
Non-local quark models represent a step towards a more realistic modeling of the QCD interactions. Nonlocal quark couplings are natural in the context of many approaches to low-energy quark dynamics: i.e. instanton liquid model, Schwinger-Dyson resummation techniques, etc. Also in lattice QCD.

Several advantages over the standard local NJL model:
• Consistent treatment of anomalies
• No need to introduce sharp momentum cut-offs
• Small next-to-leading order corrections
• Successful description of meson properties at $T = \mu = 0$ \cite{Plant98, Scarpettini04, Noguera08}

Euclidean action
for two flavors

\[
S_E = \int d^4x \left\{ \left[ \bar{\psi}(x) - i \partial + m_c \right] \psi(x) \right\} - \frac{G_s}{2} j_a(x) j_a(x) + \ldots
\]

where

\[
j_a(x) = \int d^4z \ g(z) \ \bar{\psi}(x + \frac{z}{2}) \ \Gamma_a \ \psi(x - \frac{z}{2})
\]

$g(x)$: nonlocal, well behaved covariant form factors,

$\Gamma_a = (1, i\gamma_5\tau)$
Model parameters and form factor are chosen so as to obtain a good description of the vacuum and its mesonic excitations. They are determined as follows:

- Standard bosonization of the fermion theory is performed: boson fields \( \sigma \) and \( \pi_i \) are introduced.

- Mean field approximation (MFA): expansion of boson fields in powers of meson fluctuations:

\[
\sigma(x) = \bar{\sigma} + \delta\sigma(x) \\
\bar{\pi}(x) = \delta\bar{\pi}(x)
\]

- Minimization of \( S_E \) at the mean field level leads to gap equation:

\[
\bar{\sigma} = 8N_c G_S \int \frac{d^4p}{(2\pi)^4} \frac{g(p)M(p)}{p^2 + M^2(p)}
\]

where \( M(p) = m_c + g(p) \bar{\sigma} \)

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[Graph showing the comparison of Lattice (Milc data, 2006) with Gaussian and Lorentzian fits.]
Beyond the MFA: low energy meson phenomenology

\[ S_{\text{quad}}^{E} = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \left[ G^+(p^2) \delta\sigma(p) \delta\sigma(-p) + G^-(p^2) \delta\pi(p) \cdot \delta\pi(-p) \right] \]

\[ G^\pm(p^2) = \frac{1}{G_s} - 8 N_c \int \frac{d^4q}{(2\pi)^4} g^2(q) \frac{q^+ \cdot q^- + M(q^+)M(q^-)}{q^+^2 + M^2(q^+)} \frac{q^-^2 + M^2(q^-)}{q^-^2 + M^2(q^-)} \]

Pion mass from \( G^-(-m^2_\pi) = 0 \)

Pion decay constant from \[ \langle 0 \mid A_\mu^a(0) \mid \pi^b(p) \rangle = i \delta^{ab} p_\mu f_\pi \]

Consistency with ChPT results in the chiral limit:

- GT relation
- GOR relation
- \( \pi^0 \gamma\gamma \) coupling

Plant, Birse, NPA (98), Scarpettini, Gomez Dumm, NNS, PRD (04), Gomez Dumm, Grunfeld, NNS, PRD (06)
Extension to finite $T$ and $\mu$ is obtained by using Matsubara formalism

$T_c \sim 120-140$ MeV  Rather Low!

Typical NJL model  $T_c \sim 175$ MeV

Typical lattice QCD  $T_c \sim 160 – 200$ MeV

Inclusion of diquark interactions  

\[ L_P = H \ j_D(x) \ j_D(x) \]

\[ j_D(x) = \int d^4 z \ g(z) \bar{\psi}_c(x+z/2) \gamma_5 \tau_2 \lambda_2 \ \psi(x-z/2) \]

\[ \bar{\psi}_c(x) = \gamma_2 \gamma_4 \bar{\psi}^T(x) \]

Under Compact Star conditions (i.e. quark matter + e$^-$ + $\mu$ under electric and color charge neutrality  

Corresponding EoS has been used to describe Hybrid Compact Stars 

General, Gomez Dumm, NNS, PLB (01), Gomez Dumm, NNS, PRD (02)

Inclusion of diquark interactions  

Duhau, Grunfeld, NNS PRD (04)

Under Compact Star conditions (i.e. quark matter + e$^-$ + $\mu$ under electric and color charge neutrality  

Gomez Dumm, Blaschke, Grunfeld, NNS, PRD(06)

Corresponding EoS has been used to describe Hybrid Compact Stars  

Blaschke, Gomez Dumm, Grunfeld, Klahn, NNS (07)
Two flavors nonlocal models with Polyakov loop

To account for chiral restoration and quark deconfinement we will work with a non-local model coupled to the Polyakov. Specifically for the quark sector we will use

\[
S_E = \int d^4x \left\{ \bar{\psi}(x) \left( -i \not{D} + m_c \right) \psi(x) - \frac{G_s}{2} \left[ j_a(x) j_a(x) - j_P(x) j_P(x) \right] \right\}
\]

Usual scalar-pseudoscalar non-local current-current term

\[
\left[ j_0(x), \bar{j}(x) \right] = \int d^4z \, g(z) \bar{\psi} \left( x + \frac{z}{2} \right) 1, i\gamma_5 \bar{\tau} \psi \left( x - \frac{z}{2} \right)
\]

Accounts for wave function renormalization (WFR) of quark propagator

\[
j_p(x) = \int d^4z \, f(z) \bar{\psi} \left( x + \frac{z}{2} \right) \frac{i \not{D}}{2\hat{u}_p} \psi \left( x - \frac{z}{2} \right)
\]

We bosonize this action introducing, as usual, scalar and pseudoscalar boson fields. Due to the presence of the WFR term a second scalar field \( \sigma_2(x) \) has to be introduced. Next we perform the MFA approximation

\[
\sigma_1(x) = \bar{\sigma}_1 \quad ; \quad \bar{\pi}(x) = 0 \quad ; \quad \sigma_2(x) = \bar{\sigma}_2
\]

In this approximation the quark propagator is

\[
D_0(p) = \frac{Z(p)}{-p + M(p)} \quad \text{where} \quad M(p) = Z(p) \, m_c + \bar{\sigma}_1 \, g(p)
\]

\[
Z(p) = 1 - \bar{\sigma}_2 \, f(p)^{-1}
\]
Using the Matsubara formalism thermodynamical potential in the presence of the Polyakov loop (PL) reads

\[ \Omega_{MFA}(T, \mu) = -4T \sum_{c=r,g,b} \sum_{n=-\infty}^{n=\infty} \int \frac{d^3 \vec{p}}{(2\pi)^3} \ln \left| D_0 \left( \rho_{n,\vec{p}}^c \right) / T \right|^{-2} + \frac{\bar{\sigma}_1^2}{2G_S} + \frac{\dot{\bar{\sigma}}_p^2}{2G_S} + U \bar{\Phi}, T \]

with

\[ \rho_{n,\vec{p}}^c = \left[ 2n + 1 \pi T - i\mu + \phi_c \right]^2 + \vec{p}^2 \]

since in the Polyakov gauge the MFA value of the PL can be expressed as

\[ \bar{\Phi} = \frac{1}{3} \text{Tr}_c \exp i \Phi / T \]

For the PL effective potential we take

\[ \mathcal{U}(\Phi, T) = \left[ -\frac{1}{2} a(T) \Phi^2 + b(T) \ln(1 - 6 \Phi^2 + 8 \Phi^3 - 3 \Phi^4) \right] T^4 \]

\[ a(T), b(T) \] fitted to lattice QCD results. 
Ratti, Thaler, Weise (06)

In our calculations we take \( \bar{\phi}_8 = 0 \) to have real \( \Omega_{MFA} \).
In our calculations we choose Exponential or Lorentzian forms for functions $g(r)$ and $f(r)$.

Model parameters are adjusted so that at $T=\mu=0$ we reproduce empirical $f_\pi$ and $m_\pi$ and

$$\langle q\bar{q} \rangle^{1/3} = -240 \text{ MeV}$$

Suggested by QCD Sum Rules

In general, these parametrizations lead to good phenomenology for mesons (i.e. $\pi$-$\pi$ scat. parameters, etc)

Noguera, NNS (08)

Lattice results from Parappilly et al, Phys. Rev.D73(06)054504
From $\Omega_{\text{MFA}}$ we derive gap equations

$$\frac{\partial \Omega_{\text{MFA}}}{\partial \bar{\sigma}_1} = \frac{\partial \Omega_{\text{MFA}}}{\partial \bar{\sigma}_2} = \frac{\partial \Omega_{\text{MFA}}}{\partial \phi_3} = 0$$

At vanishing chemical potential we typically obtain

Both chiral restoration and deconfinement transitions are smooth crossovers.

In the absence of quark–PL coupling chiral restoration is crossover $T_{\text{ch}} \approx 130$ MeV while deconfinement transition is $1^{\text{st}}$ order with $T_{\Phi} \approx 270$ MeV

The peaks of the chiral susceptibility $\chi_{\text{ch}}$ and the PL susceptibility $\chi_{\Phi}$ occur at approximately the same $T \approx 210$ MeV

$(\bar{\sigma}_2 \approx -0.43 \text{ almost constant})$
Phase diagram

S1
$(T_{CEP}, \mu_{CEP}) = (197, 134)$

S2
$(T_{CEP}, \mu_{CEP}) = (170, 209)$

S3
$(T_{CEP}, \mu_{CEP}) = (156, 232)$
• PNJL-type models provide a simultaneous dynamical description of the DECONFINEMENT and CHIRAL cross-over transitions at $\mu=0$.

• Non-local extension of the PNJL represents a step towards a more realistic modeling of the QCD interactions. In this framework the coupling of the non-local quark effective action to the Polyakov loop tends to increase the otherwise too low value of $T_c(\mu=0)$.

• The main effect of the WFR seems to be in the prediction for the location of the CEP.

• Quark matter description obtained with this kind of effective models can be used for the determination of EOS under compact stars conditions, etc.