Exotic phases in hot npe matter

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Neutron Star crust matter: (≈ 1% da $M_*$ and 10% do $R_*$)

- neutrons, protons and electrons
- beta - equilibrium
- charge neutrality

(neutron star cross section taken from F. Weber)
The crust properties are crucial for the understanding of several astrophysical properties:

- **Stellar evolution**: For instance, the neutrino and photon transmission, the electric resistivity, the thermal conductivity, etc. will depend on the crust characteristics.
- **The presence of a crystal lattice of atomic nuclei in the crust** is very important for the modelling of radio-pulsar glitches.
- **The transport of heat from NS core to the star surface** is determined by the thermal conductivity of the outer layers of the crust.
- **The elastic properties** may have an important role in: oscillation modes, gravitational waves, etc.

The crust structure is related to the **FRUSTRATION** phenomenon:

- The word “frustration” is borrowed from condensed matter physics: in some anti-ferromagnetic materials there is a strong competition between the spin interactions.
The interaction energy among the three pairing cannot be simultaneously minimized. The system is said to be frustrated.

**In a certain region of the neutron star crust:**

- There is a strong competition between the nuclear force (short range) and the electromagnetic force (long range).
- So, a proton cannot decide between the two following possibilities:
  1) to be bound in a cluster (nucleus) in order to minimize the strong interaction energy
  2) to stay as far as possible in order to minimize the electromagnetic energy.

(frustrated spins figure taken from P. Schiffer, Nature*420*(2002)35.)
Crust model

We consider the following lattices:

- Body cubic centered (bcc) for spherical nuclei
- Bi-dimensional hexagonal for cylindrical nuclei
- Unidimensional slab for slab-like nuclei

(bcc lattice figure taken from N. Chamel)
Crystal Lattices

- Candidates for nuclear geometries: spherical, cylindric, slab, cylindric hole, spherical hole.

- For the modelling of the crust we will use the Wigner-Seitz approximation. In practice, we approach the unit cells by spheres, cylinders and slabs.

**Crust (npe) matter interaction**

- We assume for the nucleon-nucleon interaction: Walecka-like relativistic models including the $\sigma$, $\omega$, $\rho$ and $\delta$ mesons in the Hartree approximation.
- Both constant and density dependent coupling constants are considered.
- Protons and electrons interact through the Coulomb force.

**Thomas-Fermi approximation:** We use the semi classical TF approximation, i.e., we consider the npe matter inside the W-S cell as locally homogeneous. This matter is locally approximated by a Fermi gas (at Temperature $T$):

$$N_i = \int_{V_{WS}} d^3x \, \rho_i(\vec{x}) ,$$

$$\rho_i(\vec{x}) = 2 \int \frac{d^3p}{(2\pi)^3} \left( f_i^+(\vec{x}, \vec{p}) - f_i^-(\vec{x}, \vec{p}) \right) . \quad i = p, n, e$$
The distribution functions $f^{(\pm)}(\vec{x}, \vec{p})$ for particles and anti-particles are obtained through the minimization of the grand canonical potential:

$$
\Omega[f^{(+)}_i, f^{(-)}_i, \phi_0, \omega_0, b_0, \delta_0] = E - TS - \sum_{i=n,p,e} \mu_i N_i,
$$

$$
\frac{\delta \Omega}{\delta f^{(\pm)}_i(\vec{x}, \vec{p})} = \frac{\delta \Omega}{\delta \phi_0(\vec{x})} = \frac{\delta \Omega}{\delta \omega_0(\vec{x})} = \frac{\delta \Omega}{\delta b_0(\vec{x})} = \frac{\delta \Omega}{\delta \delta_0(\vec{x})} = 0
$$

with the energy density, $E = \int d^3x \, \mathcal{E}_{TF}(\vec{x})$,

$$
\mathcal{E}_{TF}(\vec{x}) = \sum_{i=n,p,e} K_i + \mathcal{E}_\sigma + \mathcal{E}_\omega + \mathcal{E}_\delta + \mathcal{E}_\rho + \epsilon_{COUL}(\rho_p(\vec{x}), \rho_e(\vec{x}))
$$

and the entropy, $S$, is given by:

$$
S = -2 \sum_{i=n,p,e} \int d^3x \, \int \frac{d^3p}{(2\pi)^3} \left[ f^{(+)}_i(\vec{x}, \vec{p}) \ln f^{(+)}_i(\vec{x}, \vec{p}) + (1 - f^{(+)}_i(\vec{x}, \vec{p})) \ln(1 - f^{(+)}_i(\vec{x}, \vec{p})) + ((+) \leftrightarrow (-)) \right]
$$

$$
K_i = \frac{1}{\pi^2} \int dp \, p^2 \sqrt{p^2 + M_i^*(\vec{x})} \left( f^{(+)}_i + f^{(-)}_i \right), \quad i = p, n
$$
\[ \mathcal{L} = \sum_{i=p,n} \mathcal{L}_i + \mathcal{L}_e + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_\delta + \mathcal{L}_\gamma, \]

\[ \mathcal{L}_i = \overline{\psi}_i [\gamma_\mu iD^\mu - M^*] \psi_i, \quad i = p, n, \quad \mathcal{L}_e = \overline{\psi}_e [\gamma_\mu (i\partial^\mu + eA^\mu) - m_e] \psi_e. \]

where

\[ iD^\mu = i\partial^\mu - \Gamma_\nu V^\mu - \frac{\Gamma_\rho}{2} \tau \cdot b^\mu - e \frac{1 + \tau_3}{2} A^\mu, \quad M^* = M - \Gamma_s \phi - \Gamma_\delta \tau \cdot \delta, \]

Lagrangian densities for mesons and electromagnetic field:

\[ \mathcal{L}_\sigma = \frac{1}{2} \left( \partial_\mu \phi \partial_\mu \phi - m_\phi^2 \phi^2 \right), \quad \mathcal{L}_\omega = \frac{1}{2} \left( -\frac{1}{2} \Omega_\mu\nu \Omega^{\mu\nu} + m_\omega^2 V_\mu V^\mu \right), \]

\[ \mathcal{L}_\rho = -\frac{1}{4} B_{\mu\nu} \cdot B^{\mu\nu} + \frac{m_\rho^2}{2} b_\mu \cdot b^\mu, \quad \mathcal{L}_\delta = \frac{1}{2} (\partial_\mu \delta \partial_\mu \delta - m_\delta^2 \delta^2), \quad \mathcal{L}_\gamma = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \]

\[ \Omega_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu, \quad B_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \]

\[ \Gamma_i(\rho) = \Gamma_i(\rho_0) h_i(x), \quad x = \rho / \rho_0 \]
As a result of the extremun condition one obtains:

\[ f_i^{(\pm)}(\vec{x}, \vec{p}) = \frac{1}{1 + \exp[\varepsilon_i^*(\vec{x}, \vec{p}) + \nu_i]} \quad i = n, p, e \]

where

\[ \varepsilon_i^* = \sqrt{p^2 + M_i^*(\vec{x})} \quad M_i^*(\vec{x}) = M - \Gamma_\sigma(\vec{x}) \phi_0(\vec{x}) - \Gamma_\delta(\vec{x}) \delta_0(\vec{x}) \tau_3 \quad M_e^* = m_e \]

and \( \nu_i \) is the effective chemical potential:

\[ \nu_i = \mu_i - \Gamma_\omega V_0(\vec{x}) + \frac{1}{2} \Gamma_\rho b_0(\vec{x}) \tau_3 + \Sigma_0^R + \frac{e}{2} (1 + \tau_3) A_0(\vec{x}) = \mu_p \quad \nu_e = \mu_e \]

\[ (p_{Fe}^2(\vec{x}) + m_e^2)^{1/2} - eA_0(\vec{x}) = \mu_e. \]

\[ \Sigma_0^R(\vec{x}) = \frac{\partial \Gamma_v}{\partial \rho} \rho V_0 + \frac{\partial \Gamma_\rho}{\partial \rho} \rho_3 \frac{b_0}{2} - \frac{\partial \Gamma_\sigma}{\partial \rho} \rho \phi_0 - \frac{\partial \Gamma_\delta}{\partial \rho} \rho \delta_0. \]
The equations of motion for the mesons are:

\[ (-\nabla^2 + m^2)\phi(\vec{x}) = \Gamma_s[\rho(\vec{x})]\rho_s(\vec{x}) = \Gamma_s[\rho(\vec{x})](\rho_{sp}(\vec{x}) + \rho_{sn}(\vec{x})), \]

\[ (-\nabla^2 + m^2_\delta)\delta_0(\vec{x}) = \Gamma_\delta[\rho(\vec{x})]\rho_{3}(\vec{x}) = \Gamma_\delta[\rho(\vec{x})](\rho_{sp}(\vec{x})) - \rho_{sn}(\vec{x})), \]

\[ (-\nabla^2 + m^2_\omega)V_0(\vec{x}) = \Gamma_\omega[\rho(\vec{x})]\rho(\vec{x}) = \Gamma_\omega[\rho(\vec{x})](\rho_p(\vec{x})) + \rho_n(\vec{x})), \]

\[ (-\nabla^2 + m^2_{\rho})b_0(\vec{x}) = \frac{\Gamma_{\rho}\rho(\vec{x})}{2}\rho_3(\vec{x}) = \Gamma_\omega(\rho_p(\vec{x})) - \rho_n(\vec{x})), \]

\[ -\nabla^2 A_0(\vec{x}) = e(\rho_p(\vec{x})) - \rho_e(\vec{x})). \]

**Numerical technique:**
We expand all mesonic fields in a harmonic oscillator basis (1D, 2D, 3D). So, the differential equations are transformed in matrix equations which are self-consistently solved.
We include the following W-S cells:

- sphere ⇒ spherical symmetric densities: \( \rho_i(r, \theta, \phi) = \rho_i(r) \)
- cylinder ⇒ cylindrical symmetric densities: \( \rho_i(\rho, \theta, z) = \rho_i(\rho) \)
- slab ⇒ slab symmetric densities in the x-y plane: \( \rho_i(x, y, z) = \rho_i(z) \)

The algorithm consists in calculating the minimum free energy, \( F = E - TS \), (energy) for a fixed baryonic density and temperature (\( T = 0 \)) according to the three geometries of the W-S cell:

- imposing charge neutrality

- imposing i) beta-equilibrium or ii) fixed proton fraction (\( y_p = \frac{Z}{A} \))

From the comparison between the minimum free energy associated to each one of the three geometries (3D, 2D, 1D) with the corresponding homogeneous system energy, at a fixed \( \rho_B \), we obtain the most energetically favourable configuration (i.e., the configuration that minimizes the free energy)
Results

These densities correspond to exotic structures which can be compared to: meatball (droplet), spaghetti (rod), lasagna (slab), penne (tube), swiss cheese (bubble). In the above figures $y_p=0.3$, $T=0$ and the density-dependent GDFM parametrization was used.

Typical results where the most energetically favourable phases, within the Thomas-Fermi approximation, are: droplet(meatball), slab(lasagna), rod(spaghetti). (We impose beta-equilibrium, charge neutrality and use the DDHδ approximation)

(crust structure figure taken from N. Chamel)
Phases diagram at $T = 0$ for proton fraction $y_p = 0.5$
Comparison between the homogeneous and the exotic phases energy for the npe matter using the NL3 interaction (red corresponds to the homogeneous phase).
Phases diagram at $T = 0$ and $T = 5$MeV for proton fraction $y_p=0.5$ and $y_p=0.3$.

We use the NL3 and TW interaction (red corresponds to the homogeneous phase).
Densities of the inner edge of the crust (crossing points) at $T = 0$

<table>
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<tr>
<th>model EoS</th>
<th>pasta (CP) $y_p = 0.5$</th>
<th>P (CP) $y_p = 0.5$</th>
<th>pasta (TF) $y_p = 0.5$</th>
<th>P (TF) $y_p = 0.5$</th>
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<td>0.113</td>
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CONCLUSIONS

• Exotic neutron rich phases can be formed in the neutron star crust (in the context of Thomas-Fermi approximation).

• The detailed crust structure is very model dependent.

• The transition density from exotic (pasta) to homogeneous phase can be compared with experimental constraints. So, physically unacceptable models can be discarded.

Future perspectives:

• Include surface corrections through the Extended Thomas-Fermi model.

• To consider deformed W-S cells

• Take into account the pairing effect.

2 - (NL3δ) C. Providência, A M. Santos

