Can Cold Quark Matter be Solid?

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What’s the nature of pulsars?

Normal NSs or QSs?

The answer to this question depends on non-pertubative QCD, and relates to one of the seven Millennium Prize Problems named by the Clay Mathematical Institute.
Summary

✓ What if pulsars are NSs?

- What if pulsars are QSs?
- Can Cold QM be solid?
- Conclusions

“Can QM be solid?”  http://vega.bac.pku.edu.cn/~rxxu  R.X. Xu
What if pulsars are NSs?

- Hewish et al. (1968): PSRs discovered!
- Gold (1968): Pulsars are rotating NSs

Kepler frequency in Newtonian gravity:

$$\nu_{\text{Kepler}} = \sqrt{\frac{G}{3\pi}} \bar{\rho} \approx 841 \sqrt{\frac{\bar{\rho}}{10^{14} \text{g/cm}^3}} \text{Hz.}$$

The densities of NSs (and QSs, but not WDs) could be high enough to have observed $$\nu < \nu_{\text{Kepler}}$$!

Can the NS model still work when detecting more and more PSR data (new phenomena)?
What if pulsars are NSs?

- Isolated PSRs: *non-atomic* thermal spectra

NSs: *atomic atmosphere*

**Obs:** no such feature detected with certainty!

**Ex.:** RX J1856, a DTN

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**RX J1856**

- Iron atmosphere model fit
- Solar atmosphere model fit
- Blackbody fit

*Burwitz et al. (A&A, 2001)*
What if pulsars are NSs?

- Isolated PSRs: *small* bolometric radii

![Graph showing radii and temperatures of different pulsars.](image)

**BB model:**
- CCO
- AXP
- SGR

**Pavlov et al. 2004**
IAUS218,239

**Bignami et al. 2003**

1E 1207

- $R \approx (4.5,0.8) \text{ km}$
- $T \approx (160,300) \text{ eV}$
What if pulsars are NSs?

- Isolated PSRs: *small* bolometric radii

No pulsation at $P > ~0.68$ s

Thermal comp. dominates

BB: $R = ~1$ km

H-atmosphere:

\[ \begin{aligned}
R &= (4\text{~}5) \text{ km} \\
M &< ~0.8 \, M_{\odot}
\end{aligned} \]

Pavlov & Luna (2009)
What if pulsars are NSs?

Formation of NSs: Why *supernova*?

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**Improved Models of Stellar Core Collapse and Still No Explosions: What Is Missing?**


*Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Strasse 1, D-85741 Garching, Germany*

(Received 7 March 2003; published 19 June 2003)

Two-dimensional hydrodynamic simulations of stellar core collapse are presented which for the first time were performed by solving the Boltzmann equation for the neutrino transport including a state-of-the-art description of neutrino interactions. Stellar rotation is also taken into account. Although convection develops below the neutrinosphere and in the neutrino-heated region behind the supernova shock, the models do not explode. This suggests missing physics, possibly with respect to the nuclear equation of state and weak interactions in the subnuclear regime. However, it might also indicate a fundamental problem with the neutrino-driven explosion mechanism.

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1D: failed! 2D: failed? 3D: ok???

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What if pulsars are NSs?

- Radio PSRs: *drifting* sub-pulses

PSR 0943+10

Deshpande & Rankin 1999

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RS75
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What if pulsars are QSs?

- Isolated PSRs: *non-atomic* spectra
- Isolated PSRs: *small* bolometric radii
- Formation of NSs: Why *supernova*?
- Radio PSRs: *drifting* sub-pulses

In principal, can we understand all of the different manifestations in the QS model?

“Can QM be solid?”

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What if pulsars are QSs?

- Radio PSRs: normal & slow *glitches*

“Can QM be solid?”  

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What if pulsars are QSs?

- PSRs: free or torque-induced precession

PSR B1828-11 (Stairs et al. 2000, Link & Epstein 2001)
What if pulsars are QSs?

**AXP/SGRs: Why *super-flares*?**

- 1979/03/05 of SGR 0525-66 ($P = 8.1$ s)
- 1998/08/27 of SRG 900+14 ($P = 5.16$ s)
- 2004/12/27 of SGR 1806-20 ($P = 7.45$ s)

- Energy released $\sim 10^{44-47}$ ergs
- Current model: *magnetar*!
  (e.g., Woods & Thompson 2005)
  powered by B-field decay, with
  
  \[ B \sim 10^{14-15} \text{ G} \]

\[ \tau < \sim 1 \text{ ms} !!! \]

\[ \sim 10^{46} \text{ ergs} \]

Hurley et al. 2006
What if pulsars are QSs?

- Radio PSRs: normal & slow *glitches*
- PSRs: free or torque-induced *precession*
- Thermal spectra: non-atomic *Planck-like*
- AXP/SGRs: Why *super-flares*?
- and … e.g., GRB *X-ray flares*.

In principal, these could be understood if we assume *cold* quark matter to be *solid*.

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Can cold QM be solid?

- A conjectured diagram with color-supercond.
- pQCD!
- QCD-based effective models?
- really exist??

Alford et al. RMP 80 (2008) 1455
Cold quark matter at $\rho \sim 3\rho_0$

Quark number density is $n_q \sim 1.4 \text{ fm}^{-3}$, but

$$n_u = n_d = n_s \approx 0.48 \text{ fm}^{-3} = 4.8 \times 10^{38} / \text{cm}^3.$$ 

and the distances between quarks are then

$$l_q = n_q^{-1/3} = 0.9 \text{ fm}, \quad l_u = l_d = l_s \approx n_u^{-1/3} = 1.3 \text{ fm}.$$ 

The de Broglie wavelength is:

$$\lambda_q = \frac{h}{\sqrt{2m_kT}} = 5 \times 10^3 m_{300}^{-1/2} T_6^{-1/2} \text{ fm} \gg l_q!$$

where $m_{300} = m_q/(300 \text{MeV})$, $T_6 = T/(10^6 \text{ K})$.

Quantum F-D statistics applies!

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Turning off the interaction there, we have the degenerate quark chemical potential at 0-temperature (kinematic Fermi energy):

\[ \mu_{u}^{NR} = \mu_{d}^{NR} \sim \mu_{s}^{NR} \approx \frac{\hbar^2}{2m_q} (3\pi^2)^{2/3} \cdot n_u^{2/3} = 380\text{MeV} \gg T! \]

\[ \left( \mu_{u}^{ER} = \hbar c (3\pi^2)^{1/3} \cdot n_u^{1/3} = 480\text{MeV} \gg T \right) \]

However, the strong interactions should play an important role …

• What if strong color-interaction exists?

Strong interaction ⇒ quark cluster?
Can cold QM be solid?

Using Heisenberg’s uncertainty relation, we can estimate the length scale of and interaction strength in a quark cluster:

\[ l_q \sim \frac{1}{\alpha_s \frac{\hbar c}{mc^2}} \sim 1\text{fm}/\alpha_s, \quad E_q \sim 300\alpha_s^2\text{MeV}, \]

if quarks are dressed, with mass \( \sim 300\text{MeV} \).

What about \( \alpha_s \)?
\( \alpha_s > 1 \) is possible in realistic cold QM!

Quark clustering is very likely in the approach of Dyson–Schwinger equations at least!

\[ \alpha(x) = \frac{\alpha(0)}{\ln(e + a_1 x^{a_2} + b_1 x^{b_2})} \]

where \( a_1 = 5.292 \text{GeV}^{-2a^2}, a_2 = 2.324; b_1 = 0.034 \text{GeV}^{-2b^2}, b_2 = 3.169; \]
\( x = p^2, \) and \( \alpha \) freezes at \( \alpha(0) = 2.972. \)
A solid state of quark matter?
What can q-clustering do for us?

- A stiffer EoS $\Rightarrow$ higher maximum mass
  
  $M(4U\ 1636-536) \sim 2M_{\text{sun}}$? …

- To render the quark matter solidified
  
  pulsar glitch, precession, Planckian spectrum

- Extra energy released during star-quake
  
  SGR-giant flare $\sim 10^{47}$ erg, AXP burst/glitch

- Ferro-magnetism phase transition?
  
  What’s the origin of pulsar strong B-field?

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Can cold QM be solid?

- **A stiffer** EoS $\implies$ higher maximum mass

Assuming Lennard-Jones potential between q-clusters:

$$u(r) = 4U_0 \left[ \left( \frac{r_0}{r} \right)^{12} - \left( \frac{r_0}{r} \right)^6 \right]$$

We can have a **stiffer** EoS:

$$P = P_q, \quad \rho = \frac{\epsilon_q}{c^2},$$
Can cold QM be solid?

- To render the quark matter *solidified*

Cluster lattice

- quark cluster

$$V(x)$$

- Classical solid: barrier penetration *negligible*
- Quantum solid: penetration *significant*
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✓ Conclusions
Conclusions

- Realistic cold quark matter is suggested in a *solid* state where quark *clustering* occurs.

- *Solid* QS is *necessary* to understand obs.

- A combined study should be very *essential* to know the real QCD phases:
  - Lattice QCD
  - QCD-based effective models (e.g. DSE, NJL)
  - Phenomenological models (e.g. in astrophysics)
Physicists and astrophysicists should **work together** to answer the challenging problem.

**Figure 2.** Astronomers interested in pulsar-like compact stars and physicists expert at doing with QCD now face the same challenge to know the real state of cold matter at supra-nuclear density. They have to exchange information during their work when trying to ‘dig a tunnel’ to solve the problem.
Welcome to Peking University!

- Astronomy Dept., School of Physics

- KIAA (Kavli inst. for A&A at Peking U.)
  An international platform for research of A&A
THANKS!

Comments & suggestions are welcome!

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